Causal inference in Observational Time Series

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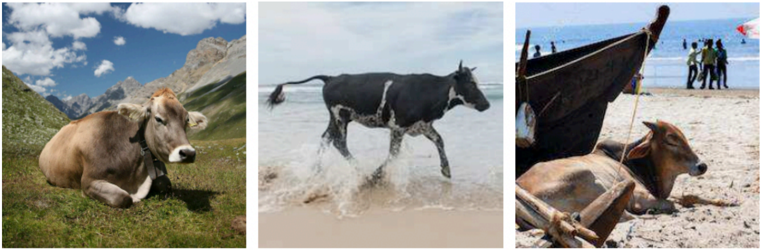
October 19, 2023

Why causality?

- Lack of generalization for ML models



(A) **Cow: 0.99**, Pasture: 0.99, Grass: 0.99, No Person: 0.98, Mammal: 0.98

(B) No Person: 0.99, Water: 0.98, Beach: 0.97, Outdoors: 0.97, Seashore: 0.97

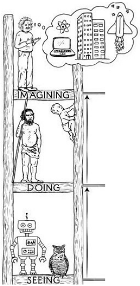
(C) No Person: 0.97, **Mammal: 0.96**, Water: 0.94, Beach: 0.94, Two: 0.94

Why causality?

- Lack of generalization for ML models
- Correlation is not causation

Why causality?

- Lack of generalization for ML models
- Correlation is not causation
- Want to answer causal questions
  - Counterfactual: Would my grandfather still be alive if he was vegetarian?
  - Intervention: How would my expected lifespan change if I become a vegetarian?
  - Association: What is the expected lifespan of somebody who is vegetarian?



Why causality?

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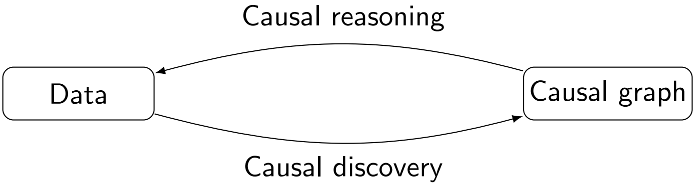
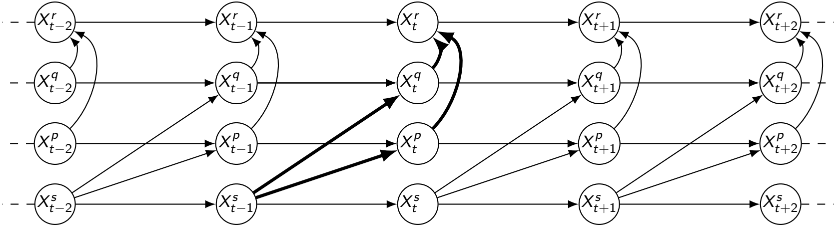


Outline

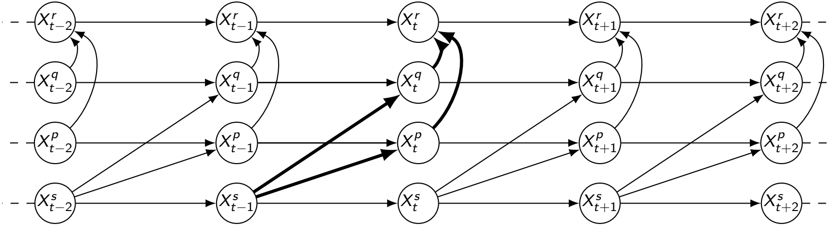
Causal graphs for time series



Full time causal graph.

A d-variate time series X
For a fixed t, each X_t is a vector $(X_t^1, \ldots, X_t^d)$,
in which X_t^p is the measurement of the pth time series at time t.

Causal graphs for time series



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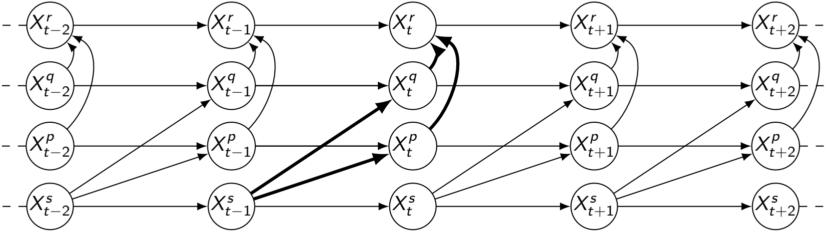
A causal graph for a multivariate time series X is said to be *consistent throughout time* if all the causal relationships remain constant in direction throughout time.

Causal graphs for time series



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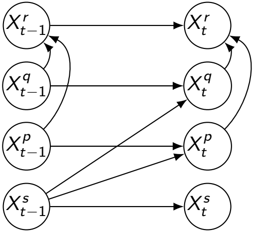
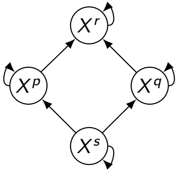
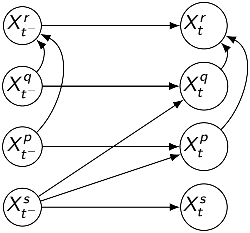


Window Causal Graph



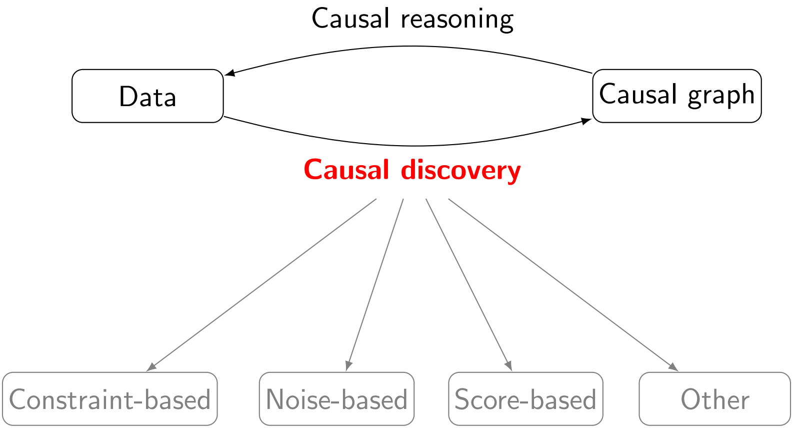
Summary Causal Graph



Extended Summary Causal Graph

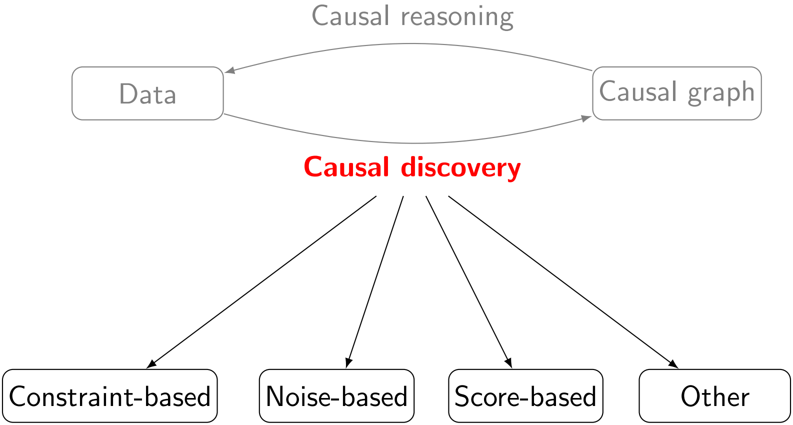
Outline

Causal discovery[1]



[1] C. K. Assaad, E. Devijver, and E. Gaussier. *Survey and evaluation of causal discovery methods for time series*. JAIR, 73, 2022.

Causal discovery[1]



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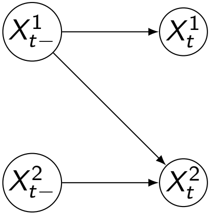
Causal discovery

Classical Assumptions

- A set of variables is said to be *causally sufficient* if all common causes of all variables are observed.
- A causal relation between two variables is said to satisfy the *temporal priority* if it is oriented in such a way that the cause occurred before its effect.
- *Causal Markov Condition*: (conditional) independence in the graph leads to (conditional) independence in the data.



$$X^1_t \perp\!\!\!\perp X^2_t \mid X^1_{t-}$$

Causal discovery

Classical Assumptions

- A set of variables is said to be *causally sufficient* if all common causes of all variables are observed.
- A causal relation between two variables is said to satisfy the *temporal priority* if it is oriented in such a way that the cause occurred before its effect.
- *Causal Markov Condition*: (conditional) independence in the graph leads to (conditional) independence in the data.
- *Minimality condition*: the graph does not contain dependencies not present in the observational data.
- *Faithfulness*: only the conditional independence relations true in the data are entailed by the Causal Markov condition applied to the graph.

$$\begin{tikzcd}
 & X_t^1 \arrow[dl, "+"'] \arrow[dr, "-"] & \\
X_t^2 \arrow[dr, "+"'] & & X_t^3 \arrow[dl, "-"] \\
 & X_t^4 &
\end{tikzcd}$$

Causal discovery

Granger Causality[2]

[2] Granger, C. (1969). Investigating causal relations by econometric models and cross-spectral methods. Econometrica, 37(3), 424–38.

Causal discovery

Granger Causality[2]

X^p Granger-causes X^q if past values of X^p provide unique statistically significant information about future values of X^q.

Pairwise Granger causality

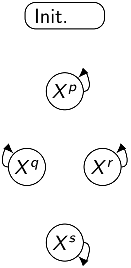


Figure: Running example: structure inferred by the pairwise Granger method (an arbitrary order has been chosen for the example).

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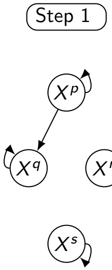

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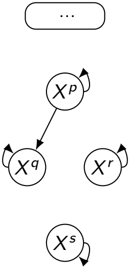

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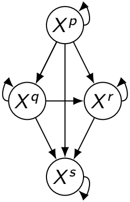
Pairwise Granger causality

Step 6



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Multivariate Granger causality

$$X_t^q = a_{q,0} + \sum_{\substack{r=1 \\ r \neq p}}^{d} \sum_{i=1}^{\tau} a_{r,i} X_{t-i}^p + \xi_t^q, \qquad \text{(mvMres)}$$

$$X_t^q = a_{q,0} + \sum_{r=1}^{d} \sum_{i=1}^{\tau} a_{r,i} X_{t-i}^r + \xi_t^q, \qquad \text{(mvMfull)}$$

Extensions

- Non-linear associations
- Nonstationnarity

Causal discovery

Constraint-based approaches

- Exploiting conditional independencies to build a skeleton
- Skeleton is oriented according to a set of rules that define constraints on admissible orientations.

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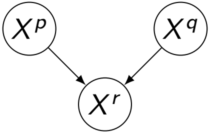
Assumptions

- Causal Markov Condition
- Faithfulness

Structures with 3 nodes

$X^p \not\perp\!\!\!\perp X^q | X^r$



Causal discovery

Constraint-based approaches

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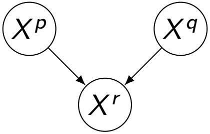

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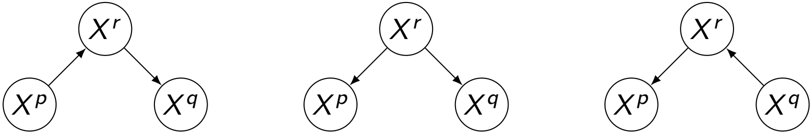

Markov equivalence class: set of DAGs that encode the same set of conditional independencies.

Main difficulty when dealing with time series: determine a good measure of (conditional) dependencies

- Measure : (partial) correlation, entropy, mutual information ...
- Estimation
- Type of data (continuous, discrete, mixed)

Representation of the time series

Optimal lag γ_{pq} and $(\lambda_{pq}, \lambda_{qp})$ the *optimal* windows:

$$\gamma_{pq}, \lambda_{pq}, \lambda_{qp} = \underset{\gamma \geq 0, \lambda_1, \lambda_2}{\operatorname{argmax}} \; h(X^q_{t:t+\lambda_2} \mid X^q_{t-1}, X^p_{t-\gamma-1}) - h(X^q_{t:t+\lambda_2} \mid X^p_{t-\gamma-1:t-\gamma+\lambda_1}, X^q_{t-1}).$$

where h denotes the entropy.

Causal discovery

Constraint-based approaches: PCMCI[3] to discover a window causal graph

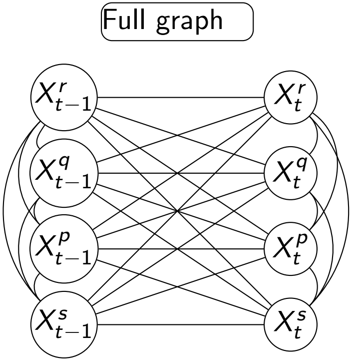

Figure: Running example: structure inferred by PCMCI with instantaneous relations.

[3] Runge, J., Nowack, P., Kretschmer, M., Flaxman, S., and Sejdinovic, D. (2019). Detecting and quantifying causal associations in large nonlinear time series datasets. Science Advances, 5(11).

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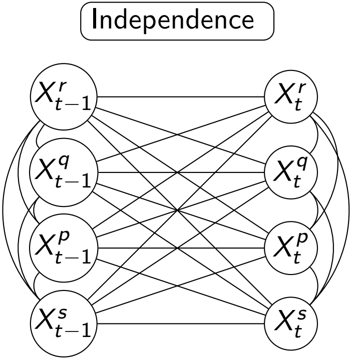

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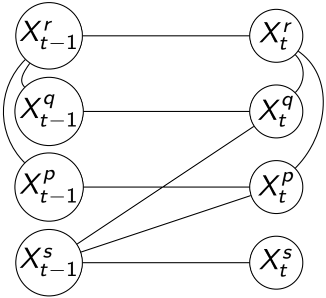
Conditional independence



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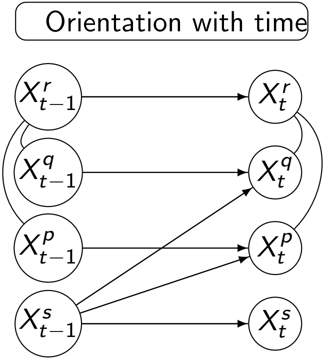

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Causal discovery

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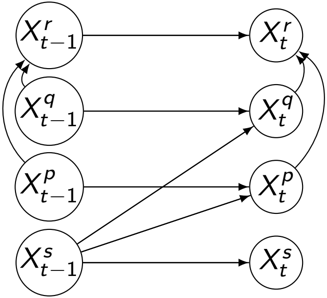
Orientation with PC



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Causal discovery

Constraint-based approaches: PCGCE[4] to discover an extended summary causal graph

Assumptions

- Causal Markov condition
- Faithfulness
- Causal sufficiency for PCGCE (but extension to FCIGCE)

[4] C. K. Assaad, E. Devijver, and E. Gaussier. *Causal Discovery of Extended Summary Graphs in Time Series*, UAI 2022

Causal discovery

Constraint-based approaches: PCGCE[4] to discover an extended summary causal graph

Assumptions

- Causal Markov condition
- Faithfulness
- Causal sufficiency for PCGCE (but extension to FCIGCE)

Measure: need a specific one due to the graph structure
Proposed Greedy Causation Entropy (GCE)

$$\mathrm{GCE}(X^p \rightarrow X^q | X^{\mathbf{Pa}}, X^{\mathbf{Pr}})$$
$$:= I(X_t^q; X_{t-\gamma:t-1}^p | X_{t-}^{Pa_1}, \cdots, X_{t-}^{Pa_l}, X_t^{Pr_1}, \cdots, X_t^{Pr_m})$$

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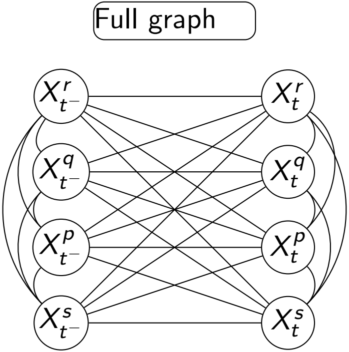
Constraint-based approaches: PCGCE[5] to discover an extended summary causal graph



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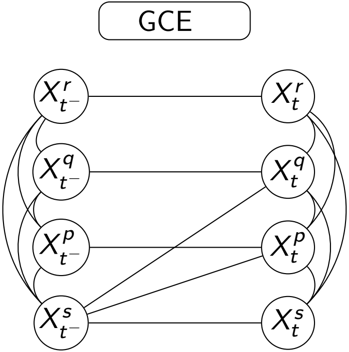

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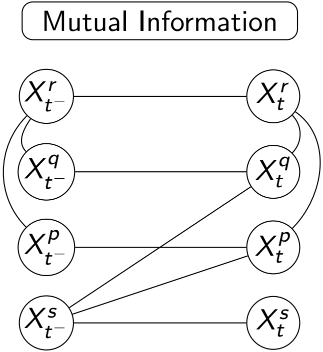

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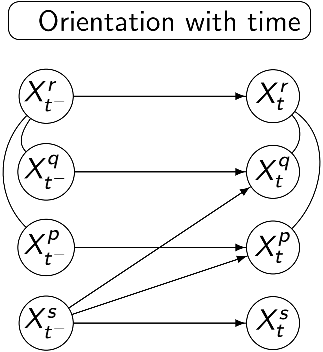

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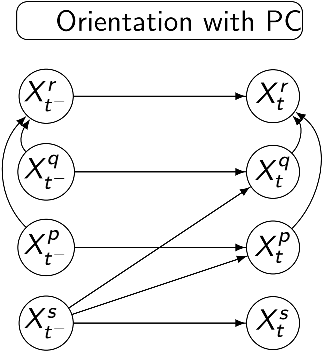

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Causal discovery

Noise-based approaches

Causal discovery

Noise-based approaches

- Causal system described by a set of equations (SEM/SCM)
- each equation explains one variable of the system in terms of its direct causes and some additional noise.

Causal discovery

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- each equation explains one variable of the system in terms of its direct causes and some additional noise.

Assumptions

- Causal Markov Condition
- Minimality

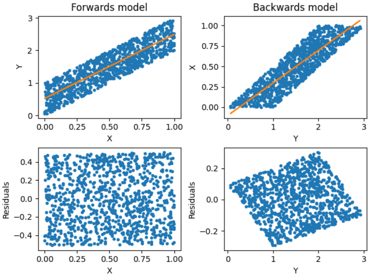
Can deal with 2 variables

Causal discovery

Noise-based approaches: Additive Noise Models

Additive noise model with nonlinear functions

$$
\begin{aligned}
X^p &= \xi^p, \\
X^q &= f_q(X^p) + \xi^q \qquad \text{with } X^p \perp\!\!\!\perp \xi_q.
\end{aligned}
$$



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Noise-based approaches: Additive Noise Models

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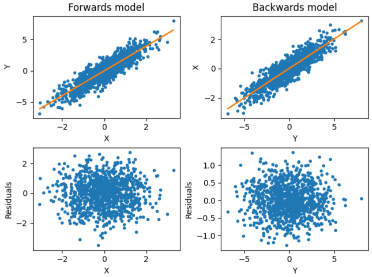

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Noise-based approaches: Additive Noise Models

Additive noise model with nonlinear functions

$$\begin{aligned} X^p &= \xi^p, \\ X^q &= f_q(X^p) + \xi^q \qquad \text{with } X^p \perp\!\!\!\perp \xi_q. \end{aligned}$$

Theorem (Identifiability of ANMs[6])

Assume that the conditional distribution of $X^q \mid X^p$ admits a smooth ANM, and that there exists $x_q \in \mathbb{R}$ such that, for almost all $x_p \in \mathbb{R}$,

$$(\log p_{\xi^q})''(x_q - f_q(x_p))f_q'(x_p) \neq 0.$$

Then, the set of log densities $\log p_X$ for which the obtained joint distribution P_{X^p,X^q} admits a smooth ANM from X^q to X^p is contained in a 3-dimensional affine space.

[6] Hoyer, P. O., Janzing, D., Mooij, J. M., Peters, J., Schölkopf, B. *Nonlinear causal discovery with additive noise models*. NeurIPS 2009

Causal discovery

Noise-based approaches: Additive Noise Models

Additive noise model with nonlinear functions

$$X^p = \xi^p,$$
$$X^q = f_q(X^p) + \xi^q \qquad \text{with } X^p \perp\!\!\!\perp \xi_q.$$

Principle (Multivariate additive noise principle)

Suppose we are given a joint distribution $P(X^1, \cdots, X^d)$. If it satisfies an identifiable Additive Noise Model such that $\{(X^p_{t-j})_{1\leq p\neq q\leq d, 0\leq j\leq\tau}, (X^q_{t-j})_{1\leq j\leq\tau}\} \rightarrow X^q$, then it is likely that $\{(X^p_{t-j})_{1\leq p\neq q\leq d, 0\leq j\leq\tau}, (X^q_{t-j})_{1\leq j\leq\tau}\}$ precedes X^q in the causal order.

Causal discovery

Noise-based approaches: VarLINGAM[6]

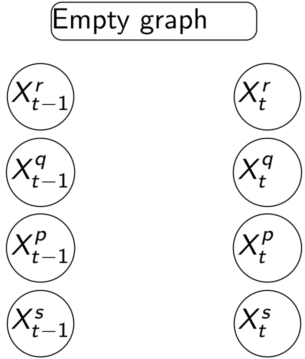

Figure: Running example: structured inferred by VarLiNGAM.

[6] Hyvarinen, A., Zhang, K., Shimizu, S., Hoyer, P. O. *Estimation of a structural vector autoregression model using non-gaussianity*. JMLR 2010

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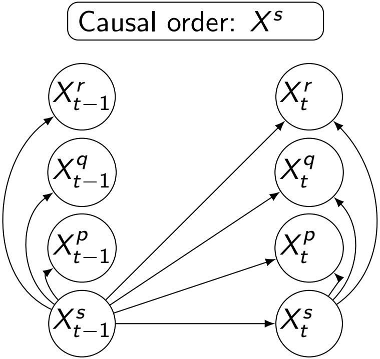

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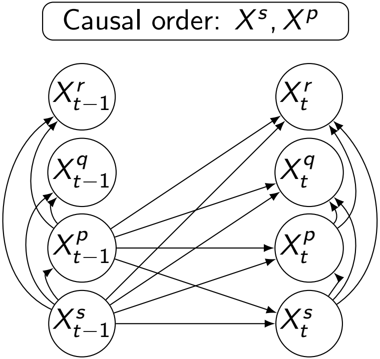

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Noise-based approaches: VarLINGAM[6]

Causal order: X^s, X^p, X^q

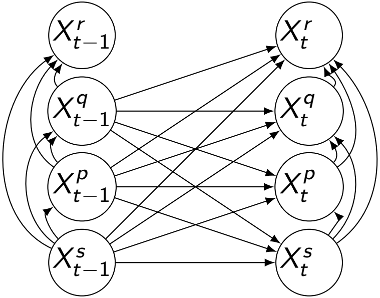


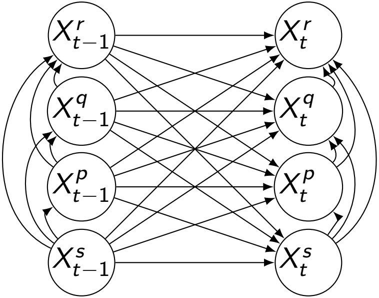
Figure: Running example: structured inferred by VarLiNGAM.

[6] Hyvarinen, A., Zhang, K., Shimizu, S., Hoyer, P. O. *Estimation of a structural vector autoregression model using non-gaussianity*. JMLR 2010

Causal discovery

Noise-based approaches: VarLINGAM[6]

Causal order: X^s, X^p, X^q, X^r



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Causal discovery

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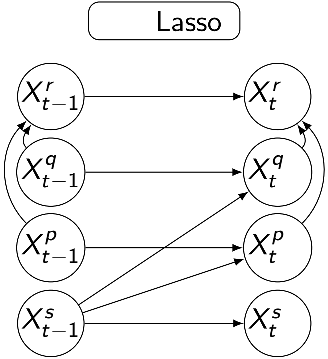

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Causal discovery

NBCB[7]: a mix between noise-based and constraint-based approaches

[7] C. K. Assaad, E. Devijver, and E. Gaussier. *A Mixed Noise and Constraint Based Approach to Causal Inference in Time Series*, ECMLPKDD 2021

Causal discovery

NBCB[7]: a mix between noise-based and constraint-based approaches

Assumptions

- Causal Markov Condition
- Adjacency faithfulness: if X^p and X^q are adjacent, then they are not conditionally independent given any subset of vertices except Xp, Xq.
- Minimality

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Causal discovery

NBCB[7]: a mix between noise-based and constraint-based approaches

Assumptions

- Causal Markov Condition
- Adjacency faithfulness: if X^p and X^q are adjacent, then they are not conditionally independent given any subset of vertices except Xp, Xq.
- Minimality

Step 1: causal ordering (additive noise model)
Last place: time series which yields the residuals that are more independent to the other time series.
Step 2: pruning to remove spurious relations based on (conditional) independence measure.

[7] C. K. Assaad, E. Devijver, and E. Gaussier. *A Mixed Noise and Constraint Based Approach to Causal Inference in Time Series*, ECMLPKDD 2021

Causal discovery

NBCB: a mix between noise-based and constraint-based approaches

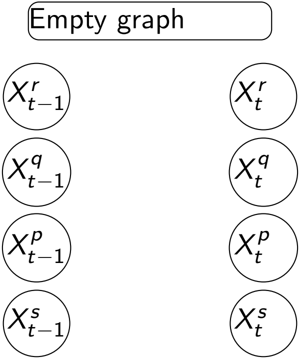


Figure: Running example: structured inferred by NBCB.

Causal discovery

NBCB: a mix between noise-based and constraint-based approaches

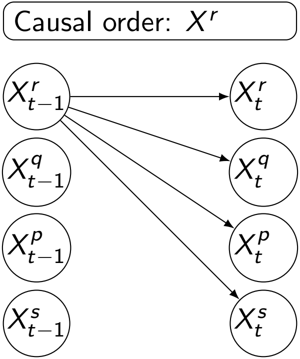


Figure: Running example: structured inferred by NBCB.

Causal discovery

NBCB: a mix between noise-based and constraint-based approaches

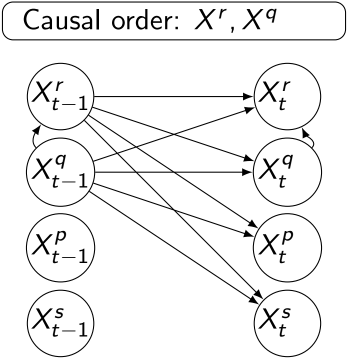


Figure: Running example: structured inferred by NBCB.

Causal discovery

NBCB: a mix between noise-based and constraint-based approaches

Causal order: X^r, X^q, X^p

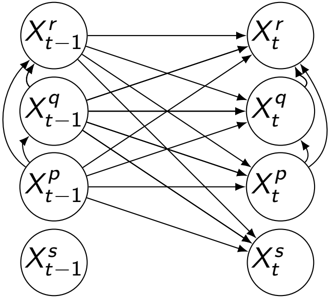


Figure: Running example: structured inferred by NBCB.

Causal discovery

NBCB: a mix between noise-based and constraint-based approaches

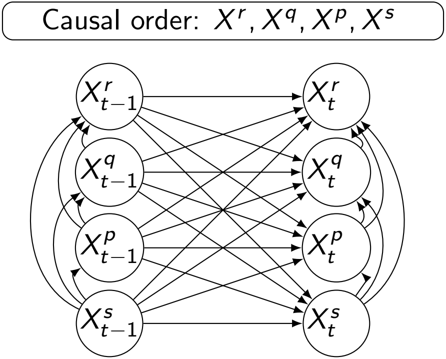


Figure: Running example: structured inferred by NBCB.

Causal discovery

NBCB: a mix between noise-based and constraint-based approaches

Conditional independence using TCE

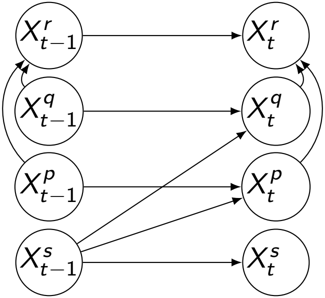
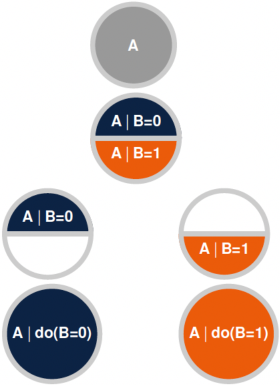


Figure: Running example: structured inferred by NBCB.

Outline

Causal reasoning
Intervention

- Population
- Sub-populations
- Conditioning
- Intervening



Causal reasoning

Problem Statement

Estimating without bias the total effect of an intervention

$$P(Y_t = y_t | do(X_{t-\gamma} = x_{t-\gamma})) = P(y_t | do(x_{t-\gamma})).$$

- Experimentation? Can be costly, unethical or even unfeasible.
- Directly from observational data? Identifiability

The total effect $P(y_t \mid do(x_{t-\gamma}))$ is said to be *identifiable* from a graph if it can be uniquely computed from the observed distribution, without any further assumption on the distribution.

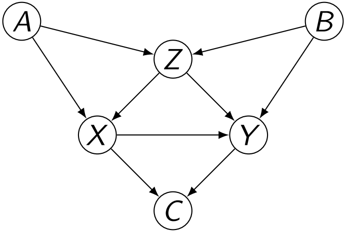
Causal reasoning

Problem Statement

Example: back-door criterion

Consider a causal graph and the total effect $P(y|do(x))$. Let $\mathcal{Z}$ a set of variables with no descendant of X and that blocks every path between X and Y that contains an arrow into X. Then, if $P(x,z) > 0$,

$$P(y|do(x)) = \sum_z P(y|x,z)P(z).$$



Assumptions: causal sufficiency, consistency throughout time.

Theorem 1[8]: Consider an FTCG $\mathcal{G}^f$ (or equivalently a WCG). The total effect $P(y_t|do(x_{t-\gamma}))$, with $\gamma \geq 0$ is identifiable in $\mathcal{G}^f$.

Theorem 2: Consider an ECG $\mathcal{G}^e$. The total effect $P(y_t|do(x_{t-\gamma}))$, with $\gamma \geq 0$ is identifiable in $\mathcal{G}^e$.

[8] Blondel et al. 2016,Shpitser et al. 2008

Causal reasoning

Identifiability in SCG[9]

Assumptions: causal sufficiency, consistency throughout time.

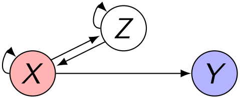
Theorem 3: Consider an SCG $\mathcal{G}^s = (\mathcal{V}^s, \mathcal{E}^s)$. The total effect $P(y_t | do(x_{t-\gamma}))$, with $\gamma \geq 0$, is not identifiable if and only if $X \in Anc(Y, \mathcal{G}^s)$ and one of the following holds:

- $\gamma \neq 0$ and $Cycles^{>}(X, \mathcal{G}^s \backslash \{Y\}) \neq \emptyset$, or
- there exists a σ-active back-door path $\pi^s = \langle V^1 = X, \cdots, V^n = Y \rangle$ such that $\langle V^2, \cdots, V^{n-1} \rangle \subseteq Desc(X, \mathcal{G}^s)$ and one of the following holds:
  - $n > 2$, ie $\langle V^2, \cdots, V^{n-1} \rangle \neq \emptyset$, or
  - $\gamma \neq 1$, or
  - $\gamma = 1$, $n = 2$ and $Cycles(Y, \mathcal{G}^s \backslash \{X\}) \neq \emptyset$.

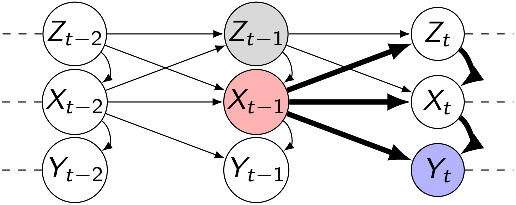
[9]A. Meynaoui et al., *Identifiability of total effects from abstractions of time series causal graphs*, submitted

Causal reasoning

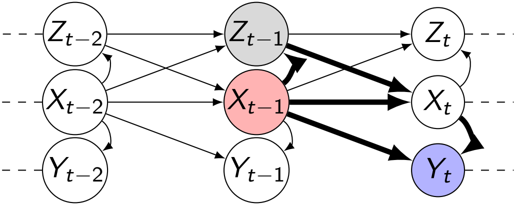
Identifiability in SCG: non identifiable example 1



An SCG $\mathcal{G}_1^s$ and the total effect $P(y_t|do(x_{t-1}))$.



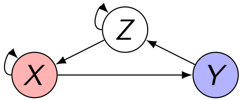
An FTCG compatible with the SCG $\mathcal{G}_1^s$.



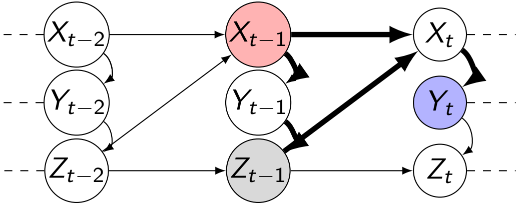
Another FTCG compatible with the SCG $\mathcal{G}_1^s$.

Causal reasoning

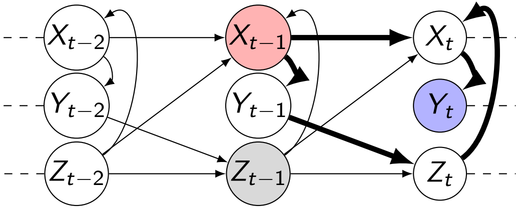
Identifiability in SCG: non identifiable example 2



An SCG $\mathcal{G}_2^s$ and the total effect $P(y_t|do(x_{t-1}))$.



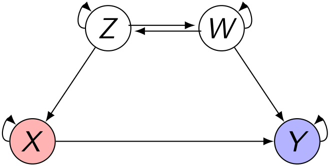
An FTCG compatible with the SCG $\mathcal{G}_2^s$.



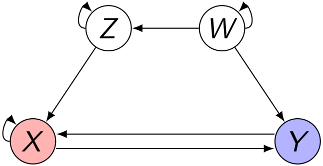
Another FTCG compatible with the SCG $\mathcal{G}_2^s$.

Causal reasoning

Identifiability in SCG: identifiable examples



$P(y_t | do(x_{t-\gamma}))$



$P(y_t | do(x_{t-1}))$

Outline

Conclusion and perspectives

- Which causal graph do we want to infer?
- The representation of time series is essential (windows - lags)
- Many families to discover causal graph for time series (also score-based, logic-based, topology-based, difference-based)
- Hybrid methods can take benefit of several worlds

Conclusion and perspectives

- Which causal graph do we want to infer?
- The representation of time series is essential (windows - lags)
- Many families to discover causal graph for time series (also score-based, logic-based, topology-based, difference-based)
- Hybrid methods can take benefit of several worlds

- Stationarity assumption for time series? Ongoing work with L. Zan
- Estimation of total effects?
- Counterfactual analysis?

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